SAMPLING PLANS AND ACCEPTANCE SAMPLING

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All figures are taken from(unless otherwise mentioned)
Introduction to Statistical process Control
Douglas. C Montgomery
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ACCEPTANCE SAMPLING

- Acceptance sampling is a method used to accept or reject product lots based on a random sample of the product.
- The purpose of acceptance sampling is to sentence lots (accept or reject) (also known as lot disposition) rather than to estimate the quality of a lot.
- Acceptance sampling plans do not improve quality, directly.
- The nature of sampling is such that acceptance sampling will accept some lots and reject others even though they are of the same quality.
- The most effective use of acceptance sampling is as an auditing tool to help ensure that the output of a process meets requirements.

Situations where acceptance sampling is likely to be useful

1. When testing is destructive.
2. When the cost of 100% inspection is extremely high.
3. When 100 % inspection is not technologically feasible or would require so much calendar time that production scheduling would be seriously impacted.
4. When there are many items to be inspected and the inspection error rate is sufficiently high that 100% inspection might cause a higher percentage of defective.
5. When the supplier has an excellent quality history, and some reduction in inspection from 100% is desired, but the supplier's process capability is sufficiently low as to make no inspection an unsatisfactory alternative.
6. When there are potentially serious product liability risks, and although the supplier's process is satisfactory, a program for continuously monitoring the product is necessary.

Types of Sampling Plans

- One major classification is by data type, **variables** and **attributes**
- Another is based on the number of samples required for a decision :
  - **Single-sampling plans**
  - **Double-sampling plans**
  - **Multiple-sampling plans**
  - **Sequential-sampling plans**
- Single-, double-, multiple-, and sequential sampling plans can be designed to produce equivalent results. Factors to consider include:
  - Administrative efficiency
  - Type of information produced by the plan
  - Average amount of inspection required by plan
  - Impact of the procedure on manufacturing flow

Lot Formation and Random Sampling

- There are a number of important considerations in forming lots for inspection, including:
  - Lots should be homogeneous.
  - Larger lots are preferred over smaller ones.
  - Lots should be conformable to materials-handling systems used in both supplier and consumer facilities.

Single Sampling Plan for attributes

- Suppose that a lot of size N has been submitted for inspection. A single-sampling plan is defined by the sample size n and the acceptance number c. Thus, if the lot size is N = 10,000, then the sampling plan:
  - n=89
  - c=2
  - means that from a lot of size 10,000 a random sample of n = 89 units is inspected and the number of nonconforming or defective items d observed.

- If the number of observed defectives d is less than or equal to c =2, the lot will be accepted.
- If the number of observed defectives d is greater than 2, the lot will be rejected.

ACCEPTANCE SAMPLING

The nature of sampling is such that acceptance sampling will accept some lots and reject others even though they are of the same quality. Thus, acceptance sampling can reject "good" lots and accept "bad" lots.

Producer's Risk

- The probability of rejecting a good lot.

➢ In order to calculate this probability there must be a numerical definition as to what constitutes "good".

➢ AQL (Acceptable Quality Level) - the numerical definition of a good lot.

The ANSI/ASQC standard describes AQL as "the maximum percentage or proportion of nonconforming items or number of nonconformities in a batch that can be considered satisfactory as a process average".

Consumer's Risk

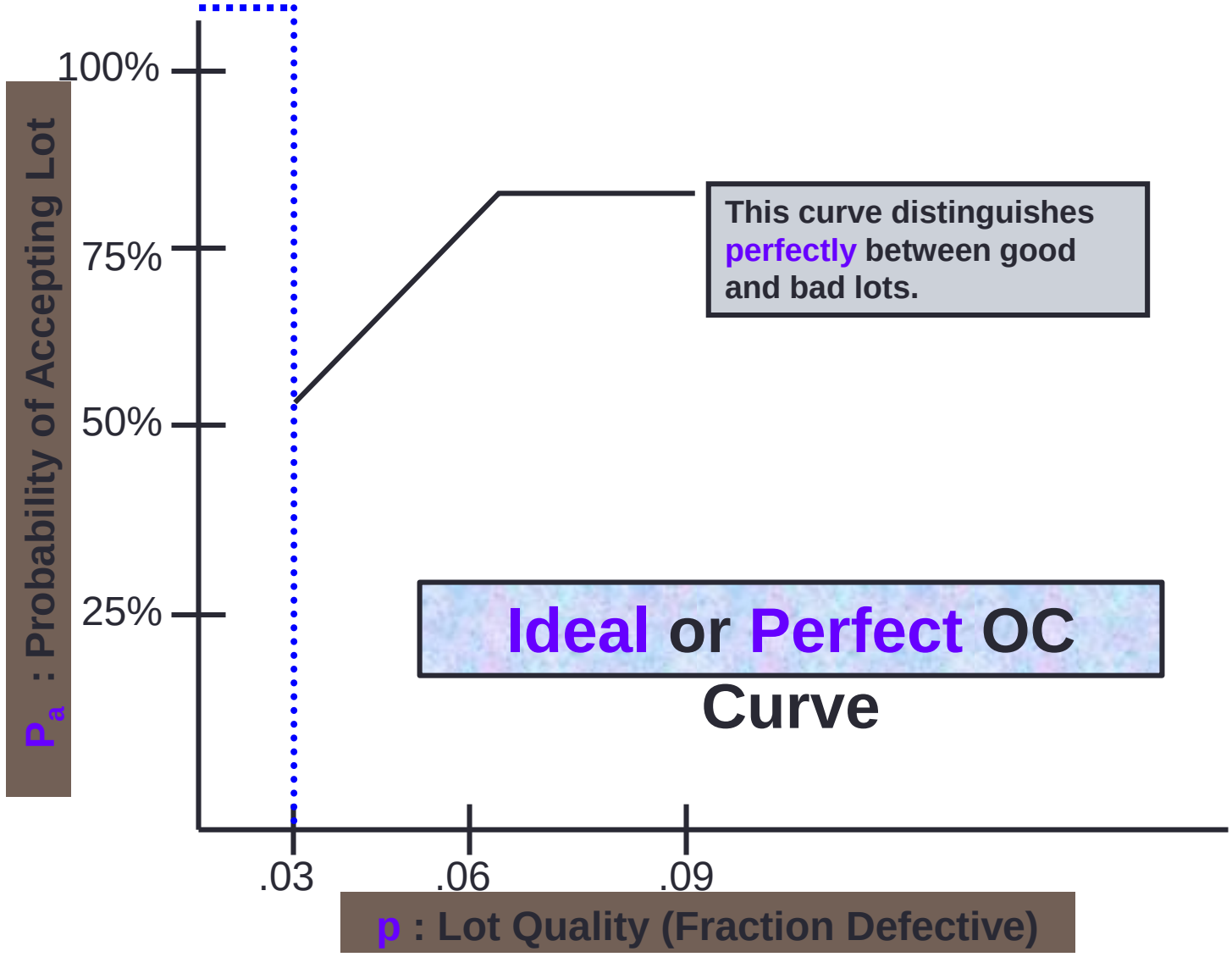
- The **probability** of **accepting** a **bad lot**.
- In order to calculate this **probability** there must be a **numerical definition** as to what **constitutes "bad".**
- **LTPD** (Lot Tolerance Percent Defective) or **RQL** (Rejectable Quality Level) **LQL** (Limiting Quality Level)
  The **numerical definition** of a **bad** lot.
- The ANSI/ASQC standard as "the **maximum percentage** or proportion of **nonconforming items** or the number of nonconformities in a batch for which the **customer** wishes the **probability** of acceptance to be a specified **low** value.

Operating Characteristic (OC) Curve

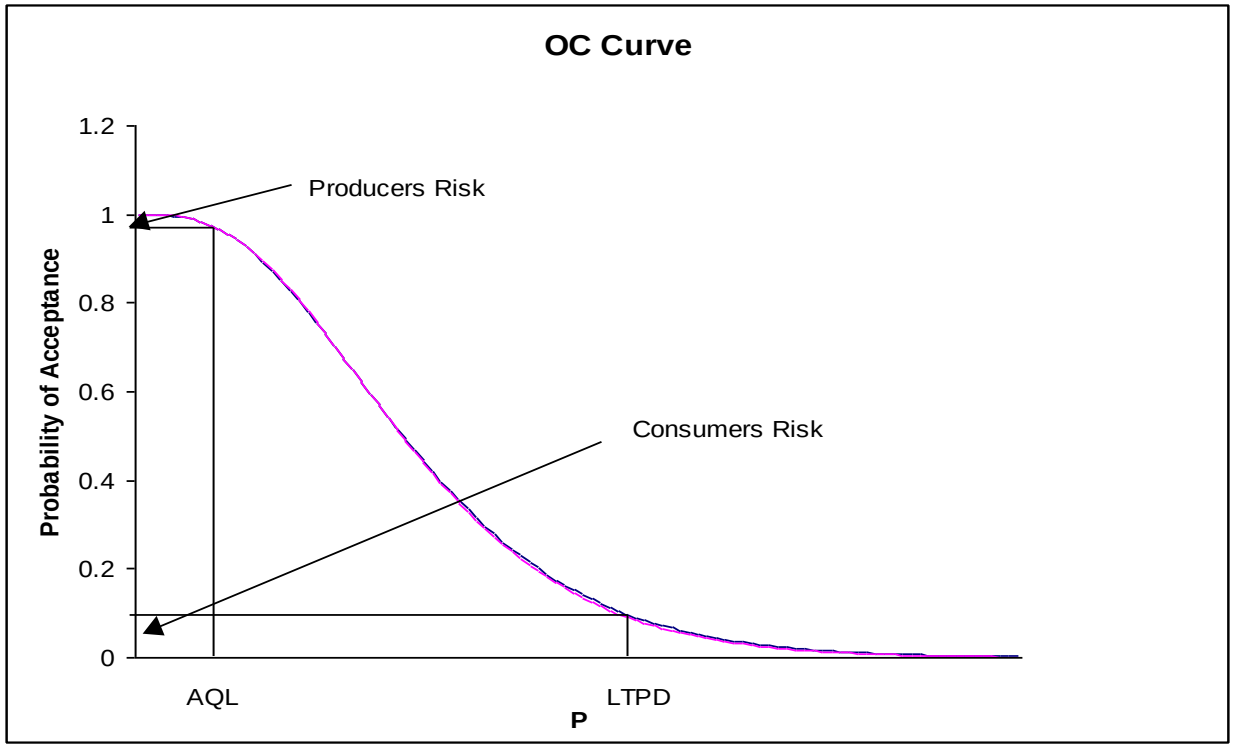
- OC Curve is a curve showing the relationship between the probability of acceptance P_a (on y-axis) and the proportion defective p of the lot (on x-axis).

- OC Curve is typically used to represent the four parameters of a sampling plan
  - Producer's Risk,
  - Consumer's Risk,
  - AQL and
  - LTPD (or RQL) of the sampling plan.

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Underlying Distribution (for 'number of defectives' in a sample)

Binomial – samples come from large lots or from a stream of lots selected at random from a process (Type B OC Curve)

Hypergeometric – from an isolated lot of finite size (Type A OC Curve)

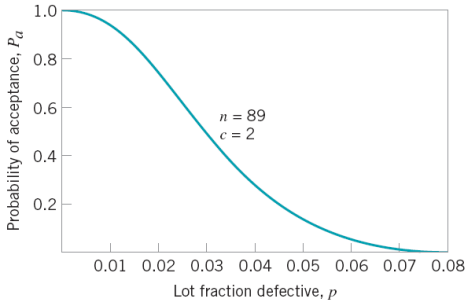
Note: If the sample is less than 20 units the binomial distribution is used to build the OC Curve; otherwise, the Poisson distribution is used

ACCEPTANCE SAMPLING

■ TABLE 15.2

Probabilities of Acceptance for the Single-Sampling Plan $n = 89$, $c = 2$

Fraction Defective, p	Probability of Acceptance, P_a
0.005	0.9897
0.010	0.9397
0.020	0.7366
0.030	0.4985
0.040	0.3042
0.050	0.1721
0.060	0.0919
0.070	0.0468
0.080	0.0230
0.090	0.0109

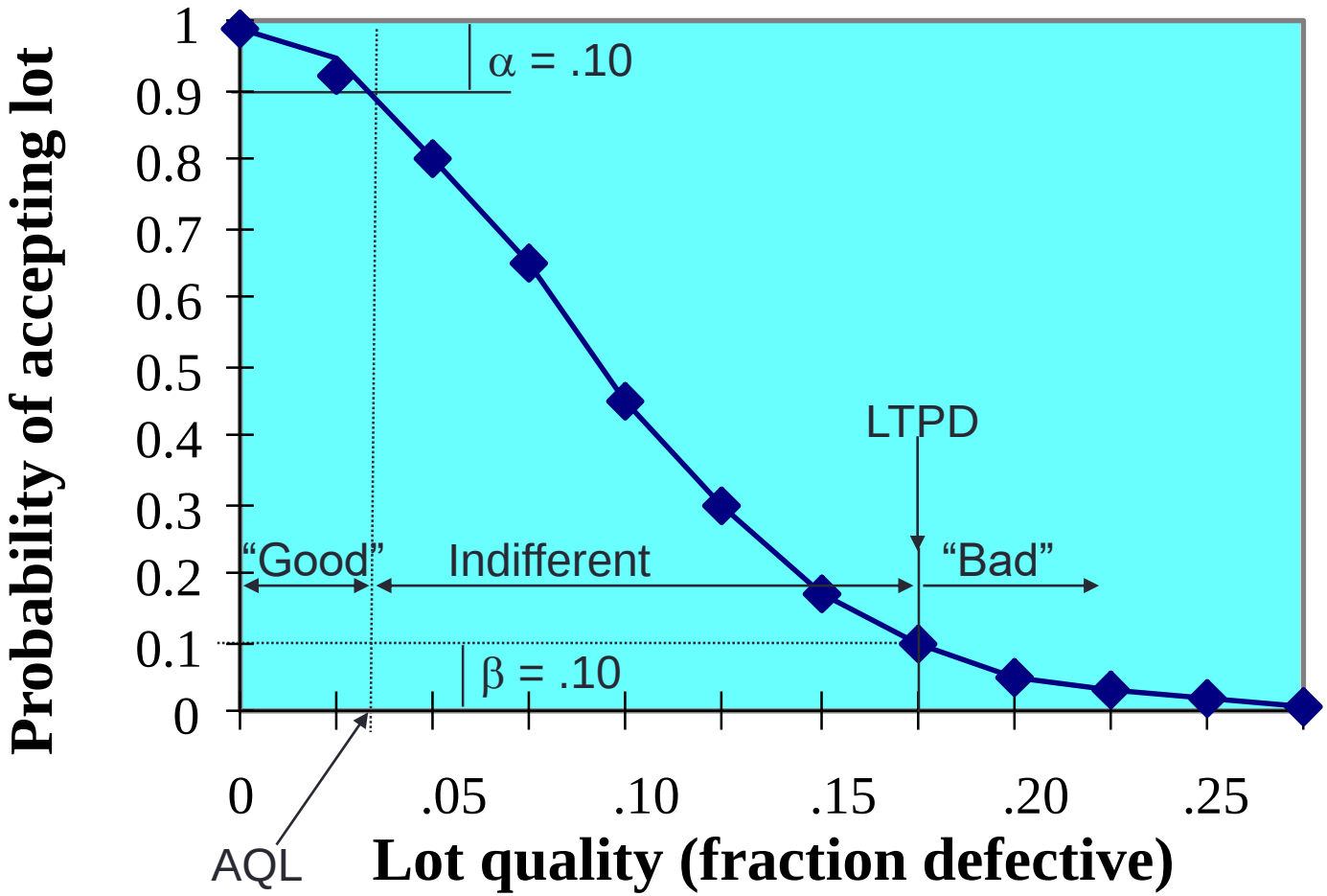


■ FIGURE 15.2 OC curve of the single-sampling plan $n = 89$, $c = 2$.

SPECIFIC POINTS ON THE OC CURVE

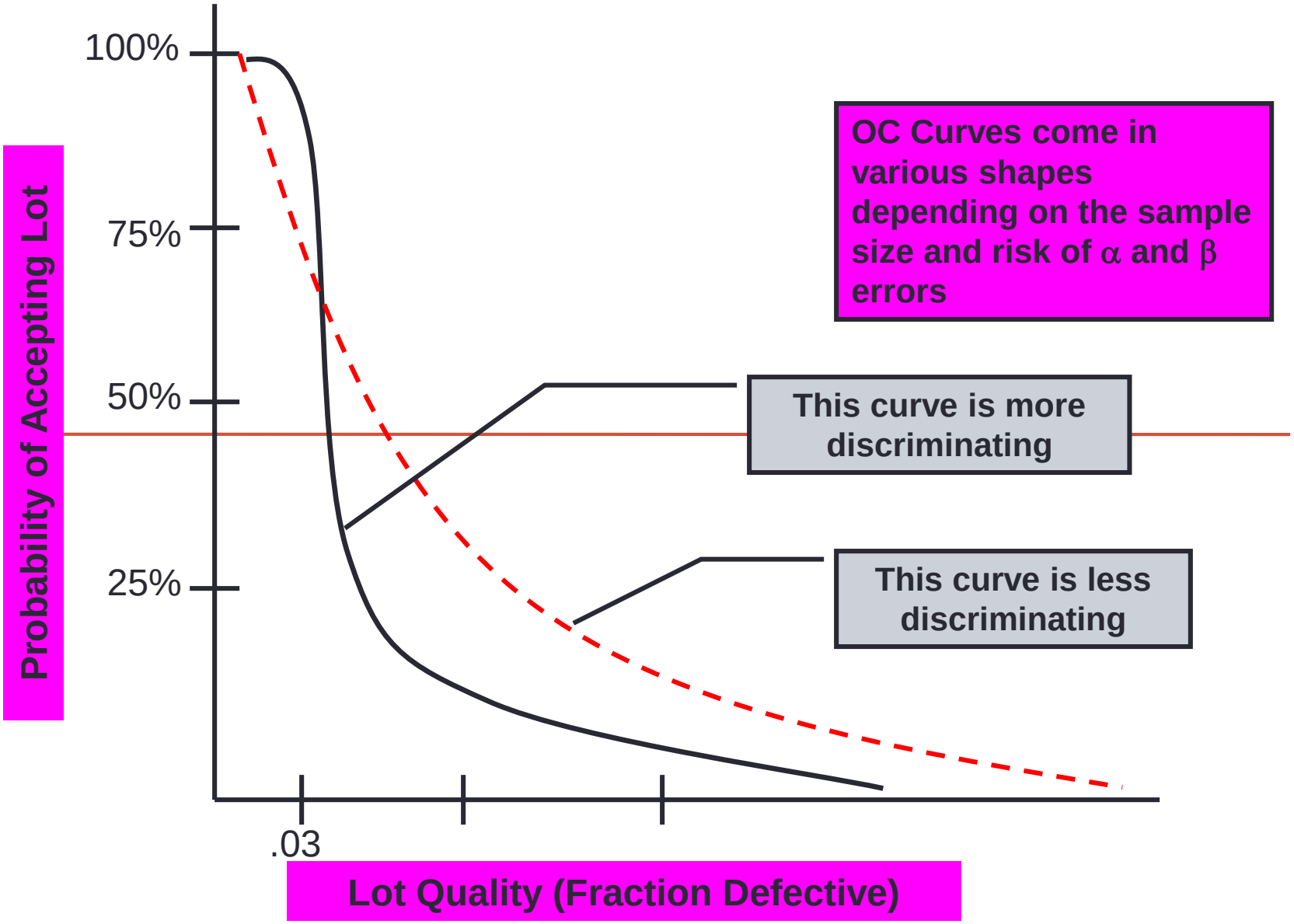
- The poorest quality level for the supplier's process that a consumer would consider to be acceptable as a process average is called the acceptable quality level (AQL)
  - AQL is a property of the supplier's manufacturing process, not a property of the sampling plan
- The protection obtained for individual lots of poor quality is established by the lot tolerance percent defective (LTPD) (Also called rejectable quality level (RQL) and the limiting quality level (LQL))
  - LTPD is a level of lot quality specified by the consumer, not a characteristic of the sampling plan
- Sampling plans can be designed to have specified performance at the AQL and the LTPD points

ACCEPTANCE SAMPLING



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ACCEPTANCE SAMPLING PLANS – OC Curve



OC Curve Calculations

- The probability of observing exactly d defectives is:

$$P\{d \text{ defectives}\} = f(d) = \frac{n!}{d!(n-d)!} p^d (1-p)^{n-d}$$

- The probability of acceptance is simply the probability that d is less than

$$P_a = P\{d \le c\} = \sum_{d=0}^{c} \frac{n!}{d!(n-d)!} p^d (1-p)^{n-d}$$

An example

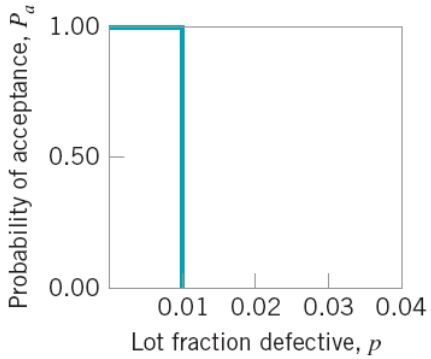
For example, if the lot fraction defective is $p = 0.01$, $n = 89$, and $c = 2$, then

$$P_a = P\{d \le 2\} = \sum_{d=0}^{2} \frac{89!}{d!(89-d)!}(0.01)^d(0.99)^{89-d}$$

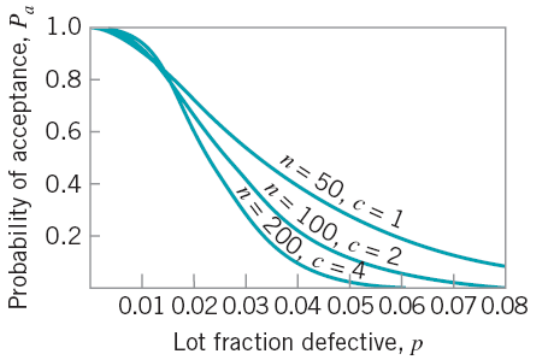
$$= \frac{89!}{0!89!}(0.01)^0(0.99)^{89} + \frac{89!}{1!88!}(0.01)^1(0.99)^{88} + \frac{89!}{2!(87)!}(0.01)^2(0.99)^{87}$$

$$= 0.9397$$

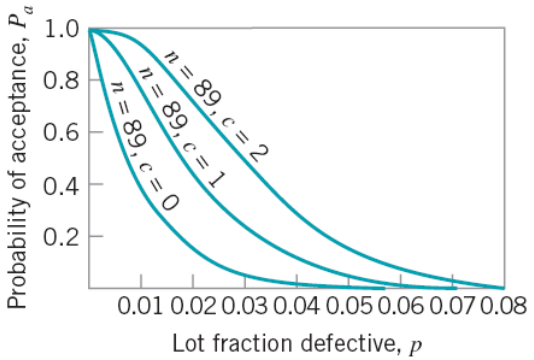
Effect of n and c on OC curve



■ FIGURE 15.3 Ideal OC curve.

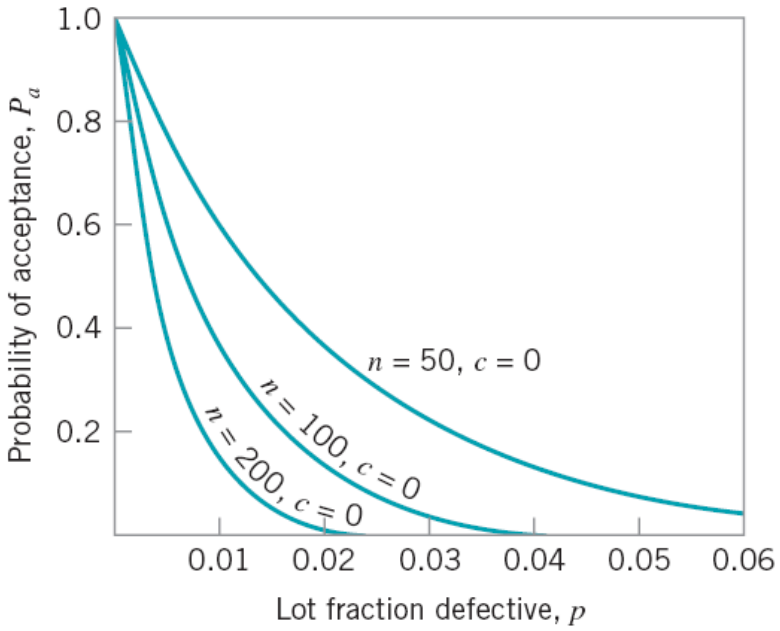


■ FIGURE 15.4 OC curves for different sample sizes.

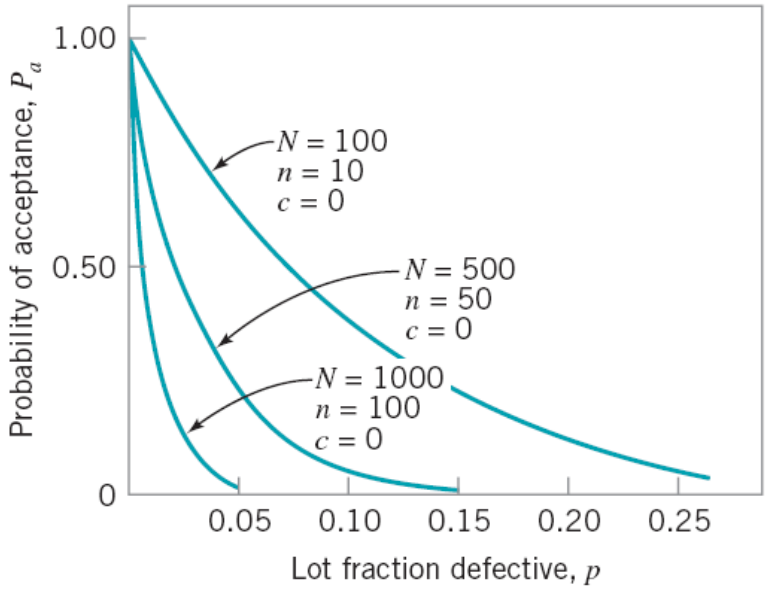


■ FIGURE 15.5 The effect of changing the acceptance number on the OC curve.

Other Aspects of OC Curve Behavior



■ FIGURE 15.7 OC curves for single-sampling plan with $c = 0$.



■ FIGURE 15.8 OC curves for sampling plans where sample size n is 10% of the lot size.

RECTIFYING INSPECTION

RECTIFYING INSPECTION

The rectifying inspection refers to the inspection programs when **100% inspection or screening** of **rejected lots** (with all discovered defective items are either **removed** or **replaced** with known good items). Consequently, average fraction defective in the stream of outgoing lots is lower than the fraction defective in the incoming lots.

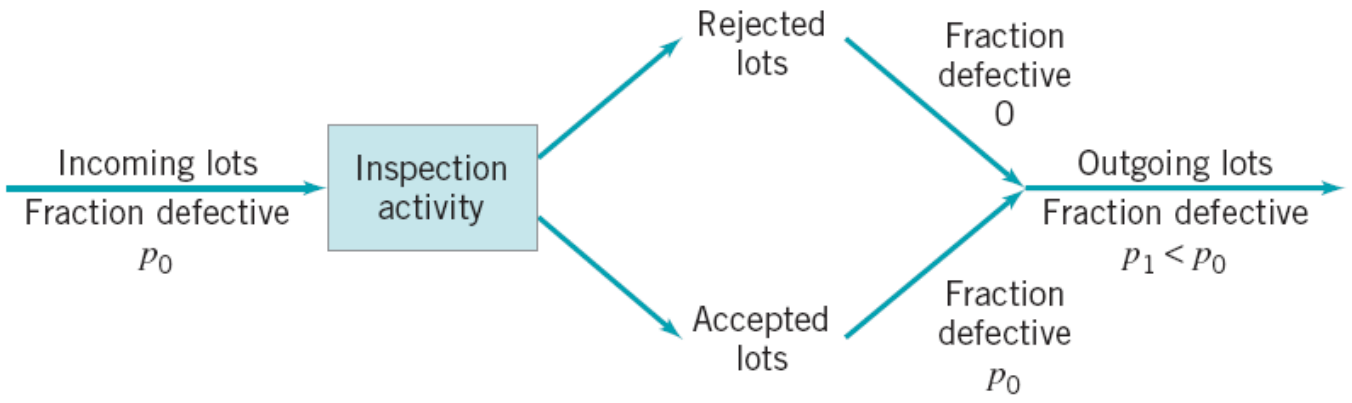


FIGURE 15.10 Rectifying inspection.

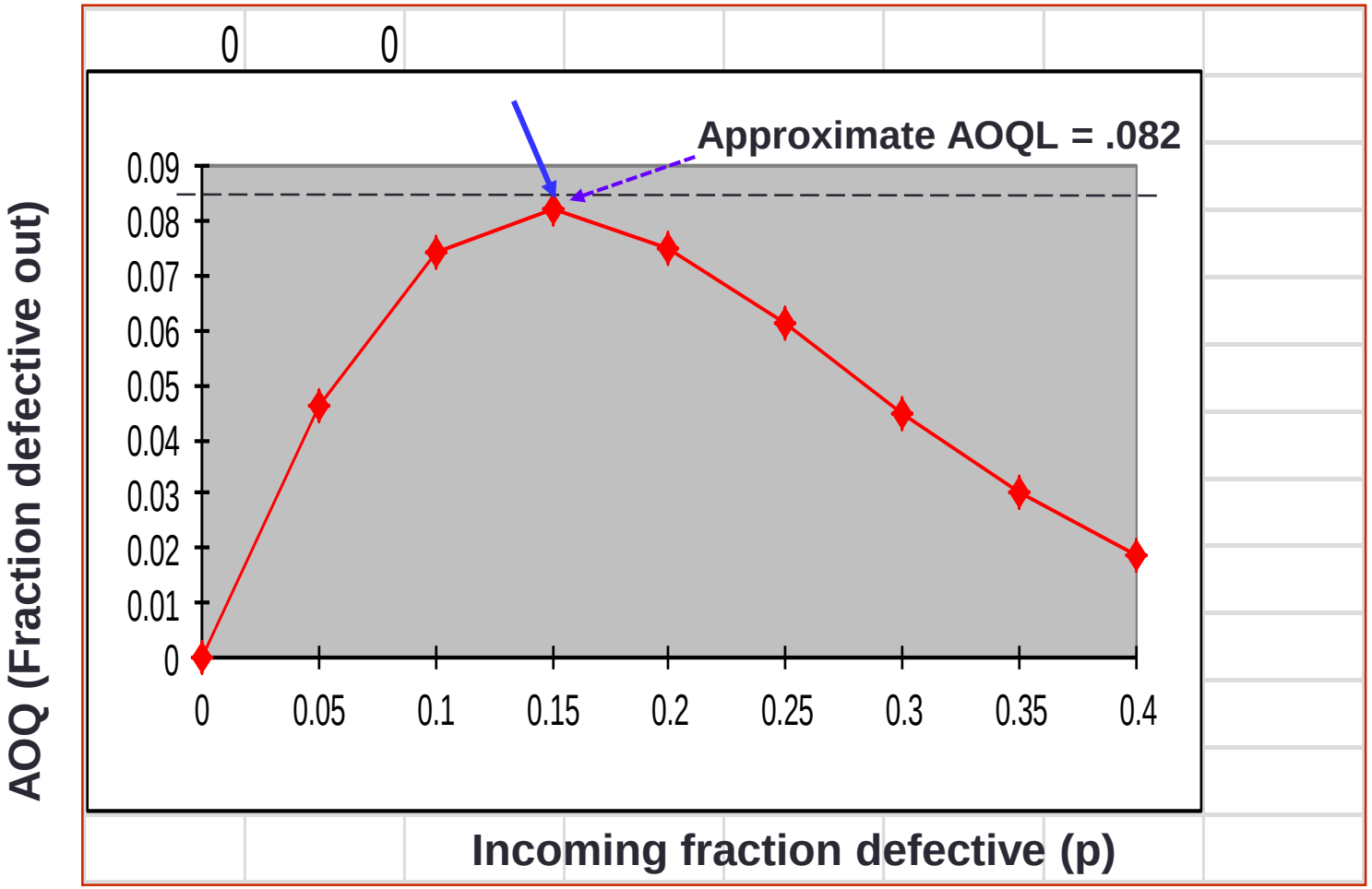
Average Outgoing Quality (AOQ)

Average of inspected lots (100%) and uninspected lots

$$AOQ = Pac \times p\left(\frac{N-n}{N}\right)$$

Pac = Probability of accepting lot
p = Fraction defective
N = Lot size
n = Sample size

Average Outgoing Quality Limit (AOQL)



AVERAGE TOTAL INSPECTION (ATI)

AVERAGE TOTAL INSPECTION (ATI)

- If the lots contain **no defectives**, no lots will be rejected, and the amount of inspection will per lot will be the sample size **n**.

- If the items are **all defective**, every lot will be submitted for 100% inspection, and the amount of inspection per lot will be the lot size **N**.

- If the lot quality (p) **0 < p < 1**, the average amount of inspection per lot will be **between the sample size n and lot size N**.

➢ If the lot is of quality p and the probability of acceptance is P_a , then the **average total inspection per lot** will be

$$ATI = n + (1 - P_a)(N - n)$$

AVERAGE TOTAL INSPECTION (ATI)

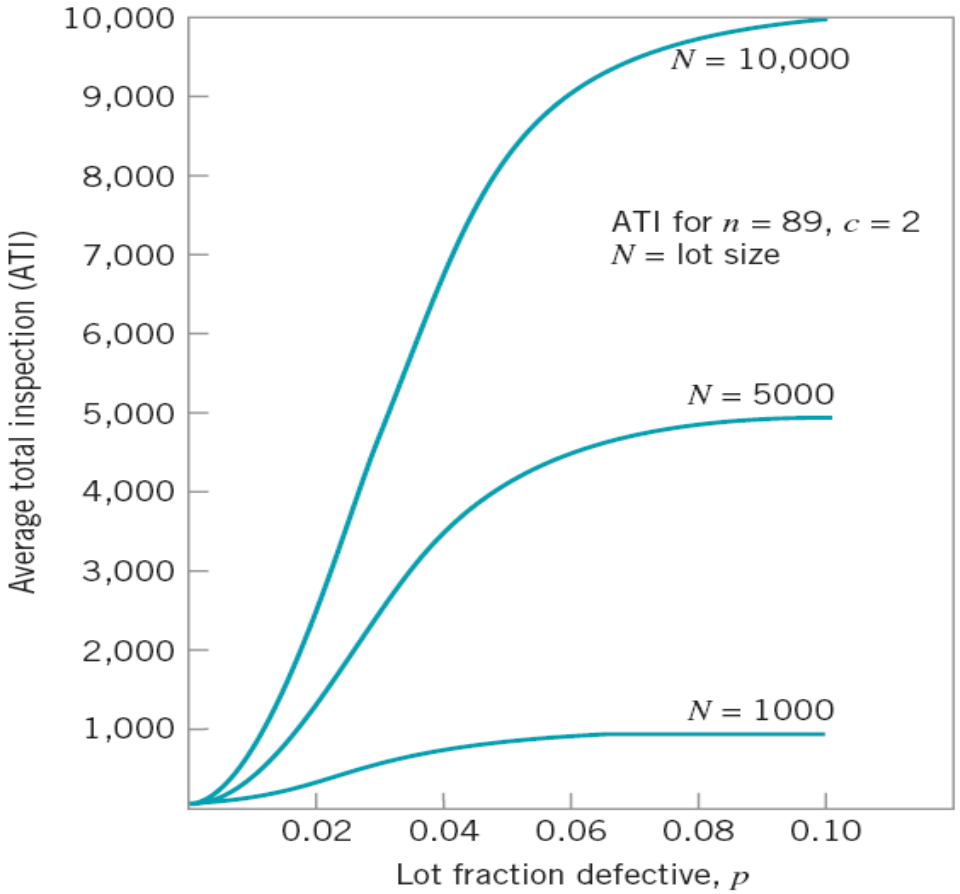


FIGURE 15.12 Average total inspection (ATI) curves for sampling plan $n = 89$, $c = 2$, for lot sizes of 1000, 5000, and 10,000.

ACCEPTANCE SAMPLING

❑ Acceptable Quality Level (AQL)

Percentage of defective items a customer is willing to accept from you (a property of mfg. process)

❑ Lot Tolerance Percent Defective (LTPD)

Upper limit on the percentage of defects a customer is willing to accept (a property of the consumer)

❑ Average Outgoing Quality (AOQ)

Average of rejected lots and accepted lots

❑ Average Outgoing Quality Limit (AOQL)

Maximum AOQ for a range of fractions defective

❑ Average Total Inspection (ATI)

Average number of units inspected per lot

Designing Sampling Plans
(Single Sampling Plans)

Operationally,

three values need to be determined before a sampling plan can be implemented (for single sampling plans):

N = (lot size) the number of units in the lot

n = (sample size) the number of units in the sample

c = (acceptance number) the maximum number of nonconforming units in the sample for which the lot will be accepted.

While there is not a straightforward way of determining these values directly given desired values of the parameters, tables have been developed.
Below is an excerpt of one of these tables.

Excerpt From a Sampling Plan Table with Producers Risk = 0.05 and Consumers Risk = 0.10			
C	*LTPD/AQL*	*n(AQL)*	*n(LTPD)*
0	44.89	0.052	2.334
1	10.946	.355	3.886
2	6.509	.818	5.324
3	4.89	1.366	6.68
4	4.057	1.97	7.992
5	3.549	2.613	9.274
6	3.206	3.286	10.535
7	2.957	3.981	11.772
8	2.768	4.695	12.996
9	2.618	5.426	14.205

Designing Sampling Plans
(Single Sampling Plans)

Three alternatives for specifying sampling plans

- **Producers Risk (1-α) and AQL specified**
- **Consumers Risk β and LTPD specified**
- **All four parameters specified**

Example 1: **Given a producers risk of .05 and an AQL of .015 determine a sampling plan**

c = 1: n = .355/.015 ~ 24

c = 4: n = 1.97/.015 ~ 131

Example 2: **Given a consumers risk of .1 and a LTPD of .08 determine a sampling plan**

c = 0: n = 2.334/.08 ~ 29

c = 5: n = 9.274/.08 ~ 116

Example 3 : **Given producers risk of .05, consumers risk of .1, LTPD 4.5%, and AQL of 1% find a sampling plan.**

Since 4.5/1 = 4.5 is between c= 3 and c = 4. Using the n(AQL) column the sample sizes suggested are 137 and 197 respectively. Note using this column will ensure a producers risk of .05. Using the n(LTPD) column will ensure a consumers risk of .1

Double Sampling Plan

In an effort to reduce the amount of inspection double (or multiple) sampling is used. Whether or not the sampling effort will be reduced depends on the defective proportions of incoming lots. Typically, four parameters are specified:

- n_1 = number of units in the first sample
- c_1 = acceptance number for the first sample
- n_2 = number of units in the second sample
- c_2 = acceptance number for both samples

✓Advantage of a double-sampling plan over single sampling is that it may reduce (??..) total amount of required inspection.

✓Suppose first sample in a double-sampling plan is smaller than for a single- sampling plan

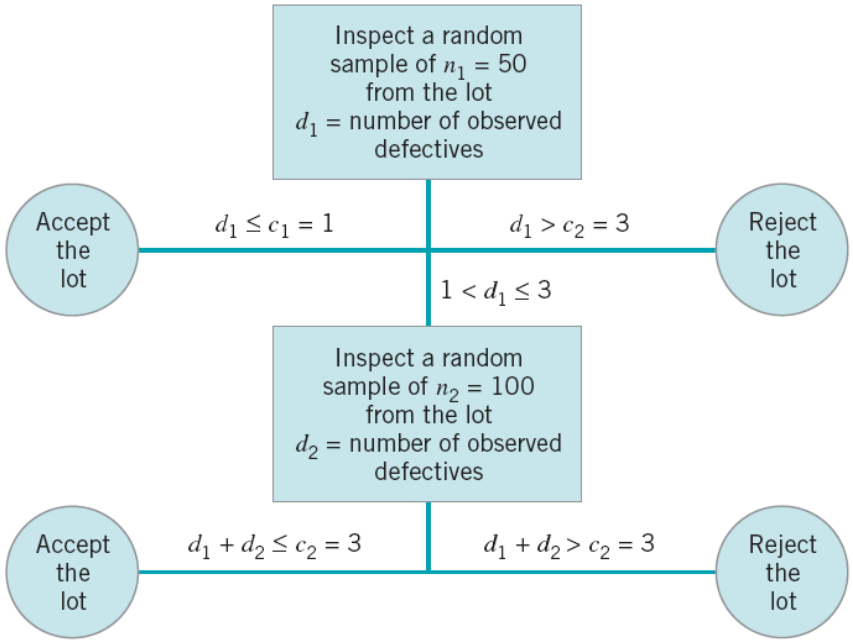
- If lot is accepted or reject on first sample, cost of inspection is lower
- Also, possible to reject a lot without completing inspection of second sample.

Double Sampling Plan

A double sampling plan proceeds as follows:

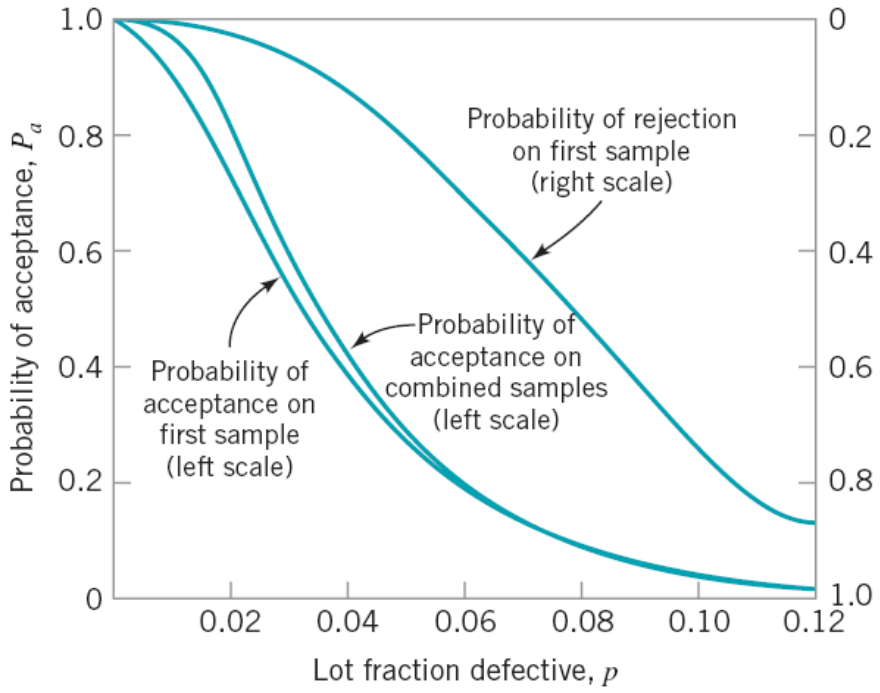
- A random sample of size n_1 is drawn from the lot.
- If the number of defective units (say d_1) $\leq c_1$ the lot is accepted.
- If $d_1 > c_2$ the lot is rejected.
- If neither of these conditions are satisfied a second sample of size n_2 is drawn from the lot.
- If the number of defectives in the combined samples $(d_1 + d_2) > c_2$ the lot is rejected. If not the lot is accepted.

Double Sampling Plan



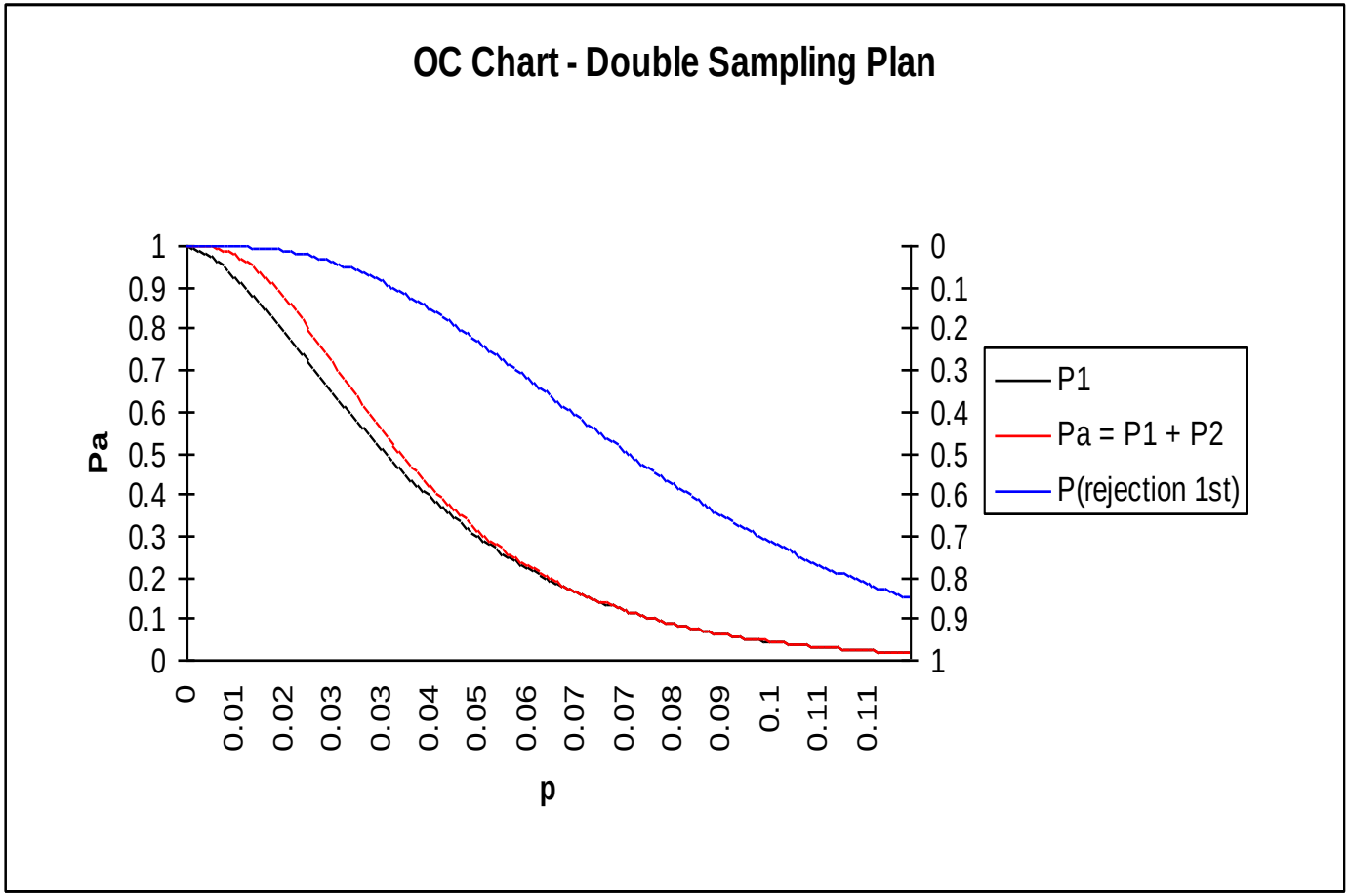
■ FIGURE 15.13 Operation of the double-sampling plan, $n_1 = 50$, $c_1 = 1$, $n_2 = 100$, $c_2 = 3$.

Double Sampling Plan
OC Curve



■ FIGURE 15.14 OC curves for the double-sampling plan, $n_1 = 50$, $c_1 = 1$, $n_2 = 100$, $c_2 = 3$.

Double Sampling Plan
OC Curve



OC Curve Calculation

- $n_1 = 50$, $c_1 = 1$, $n_2 = 100$, $c_2 = 3$. If Pa denotes the probability of acceptance on the combined samples, and P^{I}_{a} and P^{II}_{a} denote the probability of acceptance on the first and second samples,

$$P_a = P_a^{\mathrm{I}} + P_a^{\mathrm{II}}$$

- P^{I}_{a} is just the probability that we will observe $d_1 \leq c_1 = 1$ defectives out of a random sample of $n_1 = 50$ items.

$$P_a^{\mathrm{I}} = \sum_{d_1=0}^{1} \frac{50!}{d_1!(50-d_1)!} p^{d_1}(1-p)^{50-d_1}$$

Calculations

- If p = 0.05 is the fraction defective in the incoming lot

$$P_a^{\mathrm{I}} = \sum_{d_1=0}^{1} \frac{50!}{d_1!(50-d_1)!}(0.05)^{d_1}(0.95)^{50-d_1} = 0.279$$

- A second sample is drawn only if there are two or three defectives on the first sample—that is, if $c_1 < d_1 \leq c_2$.

Calculations

1. $d_1 = 2$ *and* $d_2 = 0$ or 1; that is, we find two defectives on the first sample and one or less defectives on the second sample. The probability of this is

$$P\{d_1 = 2, d_2 \le 1\} = P\{d_1 = 2\} \cdot P\{d_2 \le 1\}$$
$$= \frac{50!}{2!48!}(0.05)^2(0.95)^{48} \sum_{d_2=0}^{1} \frac{100!}{d_2!(100-d_2)!}(0.05)^{d_2}(0.95)^{100-d_2}$$
$$= (0.261)(0.037) = 0.0097$$

2. $d_1 = 3$ *and* $d_2 = 0$; that is, we find three defectives on the first sample and no defectives on the second sample. The probability of this is

$$P\{d_1 = 3, d_2 = 0\} = P\{d_1 = 3\} \cdot P\{d_2 = 0\}$$
$$= \frac{50!}{3!(47)!}(0.05)^3(0.95)^{47} \frac{100!}{0!100!}(0.05)^0(0.95)^{100}$$
$$= (0.220)(0.0059) = 0.001$$

Thus, the probability of acceptance on the second sample is

$$P_a^{\text{II}} = P\{d_1 = 2, d_2 \le 1\} + P\{d_1 = 3, d_2 = 0\}$$
$$= 0.0097 + 0.001 = 0.0107$$

The probability of acceptance of a lot that has fraction defective $p = 0.05$ is therefore

$$P_a = P_a^{\text{I}} + P_a^{\text{II}}$$
$$= 0.279 + 0.0107 = 0.2897$$

AVERAGE SAMPLE NUMBER (ASN)

AVERAGE SAMPLE NUMBER (ASN)

- In double sampling, the **size of sample** selected depends on whether or not the second sample is necessary.

- The probability of drawing a second sample **varies** with the fraction defective (p) in the incoming lot.

- With the complete inspection of the second sample, the average sample size in double sampling is equal to the (size of the first sample) x (probability that there will only one sample) + (size of the combined sample) x (probability that a second sample will be necessary)

- Therefore a general formula

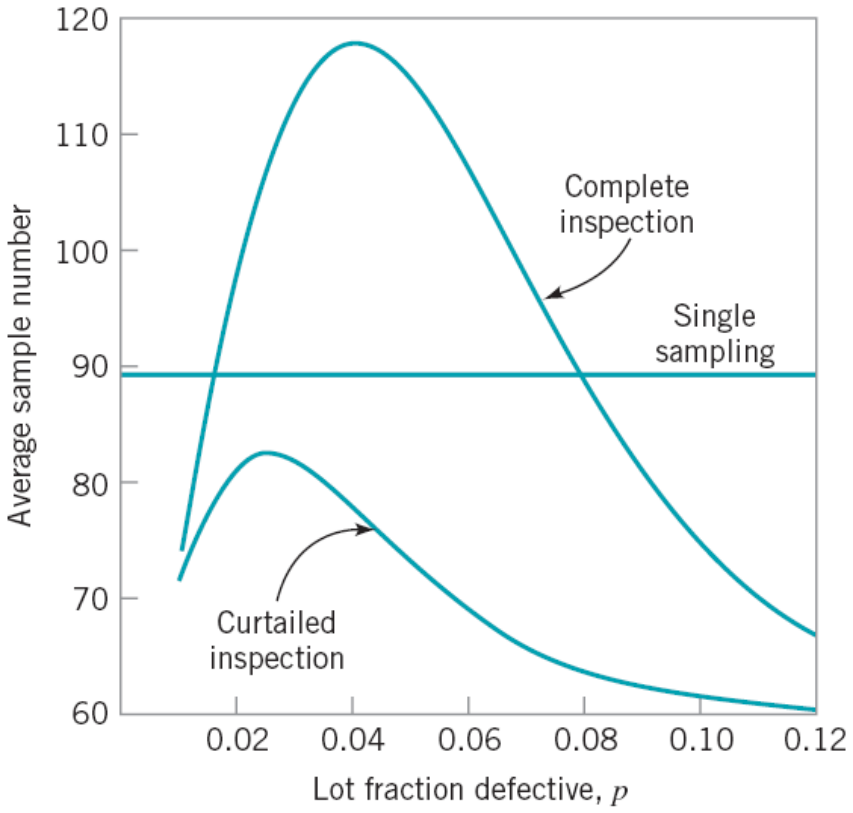
$$ASN = n_1 P_I + (n_1 + n_2)(1 - P_I) = n_1 + n_2(1 - P_I)$$

P_I is the probability of making a decision on the first sample
= P{lot is accepted on the first sample} + P{lot is rejected on the first sample}

- The plot of ASN versus p is called ASN curve.

AVERAGE SAMPLE NUMBER (ASN)

AVERAGE SAMPLE NUMBER (ASN) for Single and Double



■ FIGURE 15.15 Average sample number curves for single and double sampling.

AVERAGE SAMPLE NUMBER (ASN)

AVERAGE SAMPLE NUMBER (ASN) With Curtailement

$$\mathrm{ASN} = n_1 + \sum_{j=c_1+1}^{c_2} P(n_1, j)\left[n_2 P_L(n_2, c_2 - j) + \frac{c_2 - j + 1}{p} P_M(n_2 + 1, c_2 - j + 2)\right] \qquad (15.8)$$

Double Sampling Plan

RECTIFYING INSPECTION AND AVERAGE TOTAL INSPECTION (ATI)

Rectifying Inspection. When rectifying inspection is performed with double-sampling, the AOQ curve is given by

$$\text{AOQ} = \frac{\left[P_a^{\text{I}}(N - n_1) + P_a^{\text{II}}(N - n_1 - n_2)\right]p}{N} \qquad (15.9)$$

assuming that all defective items discovered, either in sampling or 100% inspection, are replaced with good ones. The average total inspection curve is given by

$$\text{ATI} = n_1 P_a^{\text{I}} + (n_1 + n_2)P_a^{\text{II}} + N(1 - P_a) \qquad (15.10)$$

Remember that $P_a = P_a^{\text{I}} + P_a^{\text{II}}$ is the probability of final lot acceptance and that the acceptance probabilities depend on the level of lot or process quality p.

Multiple Sampling Plan

15.3.2 Multiple-Sampling Plans

A multiple-sampling plan is an extension of double-sampling in that more than two samples can be required to sentence a lot. An example of a multiple-sampling plan with five stages follows.

Cumulative-Sample Size	Acceptance Number	Rejection Number
20	0	3
40	1	4
60	3	5
80	5	7
100	8	9

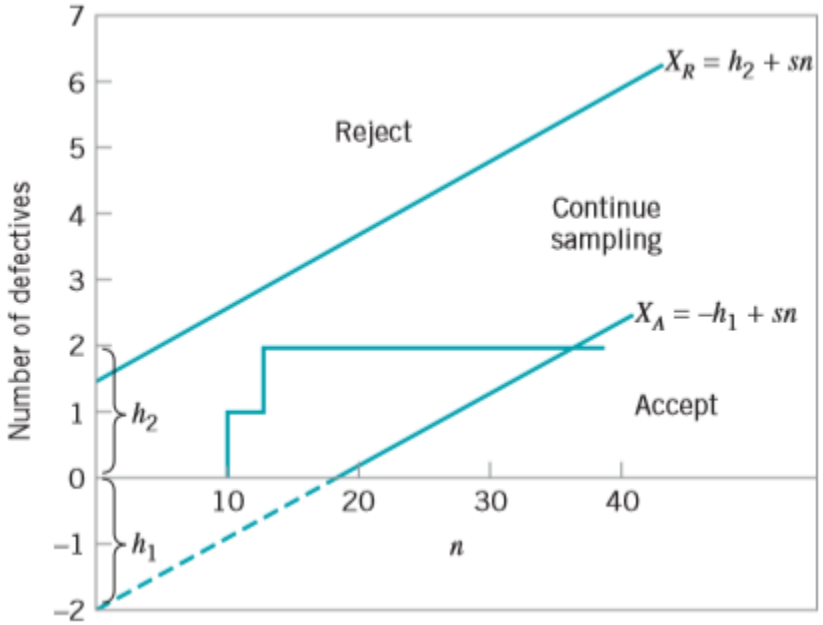
This plan will operate as follows: If, at the completion of any stage of sampling, the number of defective items is less than or equal to the acceptance number, the lot is accepted. If, during any stage, the number of defective items equals or exceeds the rejection number, the lot is rejected; otherwise the next sample is taken. The multiple-sampling procedure continues until the fifth sample is taken, at which time a lot disposition decision must be made. The first sample is usually inspected 100%, although subsequent samples are usually subject to curtailment.

Sequential Sampling Plan

15.3.3 Sequential-Sampling Plans

Sequential-sampling is an extension of the double-sampling and multiple-sampling concept. In sequential-sampling, we take a sequence of samples from the lot and allow the number of samples to be determined entirely by the results of the sampling process. In practice, sequential-sampling can theoretically continue indefinitely, until the lot is inspected 100%. In practice, sequential-sampling plans are usually truncated after the number inspected is equal to three times the number that would have been inspected using a corresponding single-sampling plan. If the sample size selected at each stage is greater than one, the process is usually called *group* sequential-sampling. If the sample size inspected at each stage is one, the procedure is usually called **item-by-item sequential-sampling.**

Example of Sequential Sampling



■ FIGURE 15.16 Graphical performance of sequential sampling.

Sequential Sampling Plan

$$X_A = -h_1 + sn \qquad \text{(acceptance line)}$$

$$X_R = h_2 + sn \qquad \text{(rejection line)}$$

where

$$h_1 = \left(\log \frac{1-\alpha}{\beta}\right) \Big/ k$$

$$h_2 = \left(\log \frac{1-\beta}{\alpha}\right) \Big/ k$$

$$k = \log \frac{p_2(1-p_1)}{p_1(1-p_2)}$$

$$s = \log\left[(1-p_1)/(1-p_2)\right] / k$$

15.4 Military Standard 105E (ANSI/ASQC Z1.4, ISO 2859)

15.4.1 Description of the Standard

Sampling procedure for inspection by attributes developed during World War II and is the most widely used acceptance-sampling system for attributes in the world today

A collection of sampling schemes; therefore an acceptance-sampling system

Provides for three types of sampling: single, double, and multiple

Primary focal point is the acceptable quality level (AQL)

- Different AQLs may be designated for different types of defects: critical, major, and minor

- Generally specified in contract or by authority responsible for sampling

Sample size is determined by lot size and by choice of inspection level

Military Standard

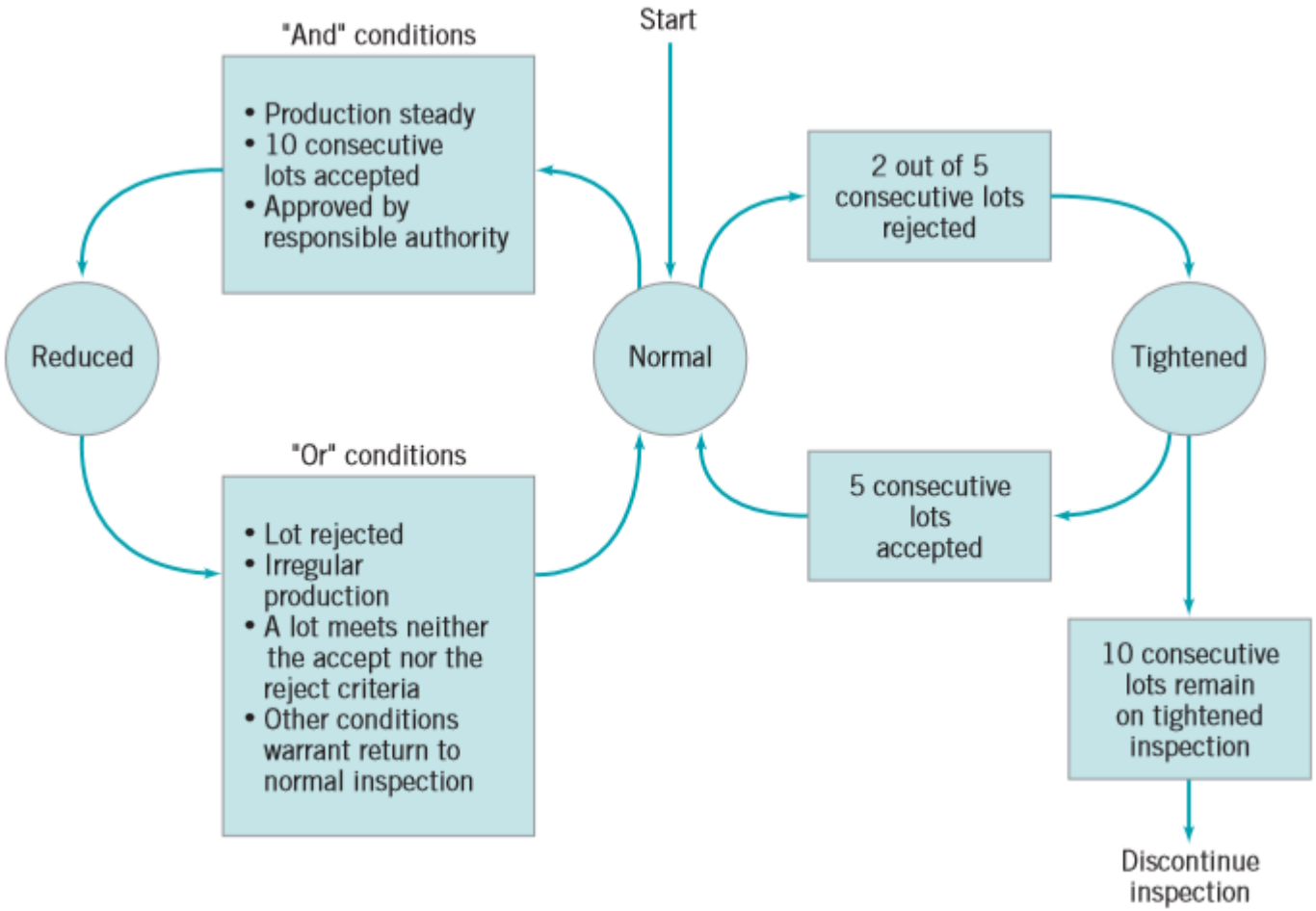
For a specified AQL and inspection level and a given lot size, MIL STD 105E provides a normal sampling plan that is to be used as long as the supplier is producing the product at AQL quality or better. It also provides a procedure for switching to tightened and reduced inspection whenever there is an indication that the supplier's quality has changed. The switching procedures between normal, tightened, and reduced inspection are illustrated graphically in Fig. 15.17 and are described next.

1. **Normal to tightened.** When normal inspection is in effect, tightened inspection is instituted when two out of five consecutive lots have been rejected on original submission.
2. **Tightened to normal.** When tightened inspection is in effect, normal inspection is instituted when five consecutive lots or batches are accepted on original inspection.
3. **Normal to reduced.** When normal inspection is in effect, reduced inspection is instituted provided all four of the following conditions are satisfied.
   - **a.** The preceding ten lots have been on normal inspection, and none of the lots has been rejected on original inspection.
   - **b.** The total number of defectives in the samples from the preceding ten lots is less than or equal to the applicable limit number specified in the standard.
   - **c.** Production is at a steady rate; that is, no difficulty such as machine breakdowns, material shortages, or other problems have recently occurred.
   - **d.** Reduced inspection is considered desirable by the authority responsible for sampling.

Military Standard

4. **Reduced to normal.** When reduced inspection is in effect, normal inspection is instituted provided any of the following four conditions are satisfied.
   - **a.** A lot or batch is rejected.
   - **b.** When the sampling procedure terminates with neither acceptance nor rejection criteria having been met, the lot or batch is accepted, but normal inspection is reinstituted starting with the next lot.
   - **c.** Production is irregular or delayed.
   - **d.** Other conditions warrant that normal inspection be instituted.
5. **Discontinuance of inspection.** In the event that 10 consecutive lots remain on tightened inspection, inspection under the provision of MIL STD 105E should be terminated, and action should be taken at the supplier level to improve the quality of submitted lots.

Military Standard- Switching Rules



15.4.2 Procedure

A step-by-step procedure for using MIL STD 105E is as follows:

1. Choose the AQL.
2. Choose the inspection level.
3. Determine the lot size.
4. Find the appropriate sample size code letter from Table 15.4.
5. Determine the appropriate type of sampling plan to use (single, double, multiple).
6. Enter the appropriate table to find the type of plan to be used.
7. Determine the corresponding normal and reduced inspection plans to be used when required.

Table 15.4 presents the sample size code letters for MIL STD 105E. Tables 15.5, 15.6, and 15.7 present the single-sampling plans for normal inspection, tightened inspection, and reduced inspection, respectively. The standard also contains tables for double-sampling plans and multiple-sampling plans for normal, tightened, and reduced inspection.

TABLE 15.4

Sample Size Code Letters (MIL STD 105E, Table 1)

	Special Inspection Levels				General Inspection Levels		
Lot or Batch Size	**S-1**	**S-2**	**S-3**	**S-4**	**I**	**II**	**III**
2 to 8	A	A	A	A	A	A	B
9 to 15	A	A	A	A	A	B	C
16 to 25	A	A	B	B	B	C	D
26 to 50	A	B	B	C	C	D	E
51 to 90	B	B	C	C	C	E	F
91 to 150	B	B	C	D	D	F	G
151 to 280	B	C	D	E	E	G	H
281 to 500	B	C	D	E	F	H	J
501 to 1200	C	C	E	F	G	J	K
1201 to 3200	C	D	E	G	H	K	L
3201 to 10000	C	D	F	G	J	L	M
10001 to 35000	C	D	F	H	K	M	N
35001 to 150000	D	E	G	J	L	N	P
150001 to 500000	D	E	G	J	M	P	Q
500001 and over	D	E	H	K	N	Q	R

Refer to Tables 15.5, -6 and -7 in the textbook on pp. 658 – 660.

Discussions

- Several points about MIL STD 105E should be emphasized:
- MIL STD 105E is AQL-oriented
- Not all possible sample sizes are possible (2,3,5,8,13,20,32,50, etc.)
- Sample sizes are related to lot sizes
- Switching rules are subject to criticism for both misswitching between inspection plans and discontinuation even though there has been no actual quality deterioration
- But a flagrant and common abuse of MIL STD 105E is failure to use the switching rules at all

15.5 The Dodge–Romig Sampling Plans

H. F. Dodge and H. G. Romig developed a set of sampling inspection tables for lot-by-lot inspection of product by attributes using two types of sampling plans: plans for lot tolerance percent defective (LTPD) protection and plans that provide a specified average outgoing quality limit (AOQL). For each of these approaches to sampling plan design, there are tables for single- and double-sampling.

Sampling plans that emphasize LTPD protection, such as the Dodge–Romig plans, are often preferred to AQL-oriented sampling plans, such as those in MIL STD 105E, particularly for critical components and parts. Many manufacturers believe that they have relied too much on AQLs in the past, and they are now emphasizing other measures of performance, such as defective parts per million (ppm). Consider the following:

AQL	Defective Parts per Million
10%	100,000
1%	10,000
0.1%	1,000
0.01%	100
0.001%	10
0.0001%	1

Thus, even very small AQLs imply large numbers of defective ppm. In complex products, the effect of this can be devastating. For example, suppose that a printed circuit board contains 100 elements, each manufactured by a process operating at 0.5% defective. If the AQLs for these elements are 0.5% and if all elements on the printed circuit board must operate for the card to function properly, then the probability that a board works is

$$P(\text{function properly}) = (0.995)^{100} = 0.6058$$

Thus, there is an obvious need for sampling plans that emphasize LTPD protection, even when the process average fallout is low. The Dodge–Romig plans are often useful in these situations.

AOQL Plans:

- **Dodge-Romig (1959) tables give AOQL sampling plans for specified AOQL values**
- **Six classes of values for process average are specified for various lot sizes**
- **Tables are available for both single and double sampling**
- **Designed so that average total inspection at a given AOQL and process average is approximately a minimum**
- **Refer to Table 15.8 for an example**

LTPD Plans:

- Dodge-Romig LTPD tables are designed so that the probability of lot acceptance at the LTPD is 0.1
- Tables are provided for various LTPD values
- Six classes of values for process average are specified for various lot sizes
- Refer to Table 14-9 for an example

Important Terms and Concepts

100% inspection
Acceptable quality level (AQL)
Acceptance-sampling plan
ANSI/ASQC Z1.4, ISO 2859
AOQL plans
Attributes data
Average outgoing quality
Average outgoing quality limit
Average sample number curve
Average total inspection
Dodge–Romig sampling plans
Double-sampling plan
Ideal OC curve
Lot disposition actions
Lot sentencing
Lot tolerance percent defective (LTPD)
LTPD plans
MIL STD 105E
Multiple-sampling plan
Normal, tightened, and reduced inspection
Operating-characteristic (OC) curve
Random sampling
Rectifying inspection
Sample size code letters
Sequential-sampling plan
Single-sampling plan
Switching rules in MIL STD 105E
Type-A and Type-B OC curves
Variables data

Guidelines for Using Acceptance Sampling

- An **acceptance-sampling plan** consists of sample size and acceptance/rejection criteria for lot sentencing
- An **acceptance-sampling scheme** is a set of procedures consisting of acceptance-sampling plans in which lot sizes, sample sizes, and acceptance/rejection criteria along with amount of 100% inspection and sampling are related
- A **sampling system** is a unified collection of one or more schemes

TABLE 15.1

Acceptance-Sampling Procedures

Objective	Attributes Procedure	Variables Procedure
Assure quality levels for consumer/producer	Select plan for specific OC curve	Select plan for specific OC curve
Maintain quality at a target	AQL system; MIL STD 105E, ANSI/ASQC Z1.4	AQL system; MIL STD 414, ANSI/ASQC Z1.9
Assure average outgoing quality level	AOQL system; Dodge–Romig plans	AOQL system
Reduce inspection, with small sample sizes, good-quality history	Chain sampling	Narrow-limit gauging
Reduce inspection after good-quality history	Skip-lot sampling; double sampling	Skip-lot sampling; double sampling
Assure quality no worse than target	LTPD plan; Dodge–Romig plans	LTPD plan; hypothesis testing